

S-Matrix Formulation of Mesoscopic Systems and Evanescent Modes

Sheelan Sengupta Chowdhury¹, P. Singha Deo¹, A. M. Jayannavar² and M. Manninen³

¹*Unit for Nanoscience and Technology,*

S. N. Bose National Centre for Basic Sciences,

Sector-III, Block-JD, Salt Lake, Kolkata-700098, India

²*Institute of Physics, Sachivalaya marg, Bhubaneswar 750015, India*

³*Nanoscience center, Department of Physics,*

University of Jyvaskyla, PO Box 35, 40100 Jyvaskyla, Finland

Abstract

The Landauer-Buttiker formalism is an important formalism to study mesoscopic systems. Its validity for linear transport is well established theoretically as well as experimentally. Akkermans et al [Phys. Rev. Lett. **66**, 76 (1991)] had shown that the formalism can be extended to study thermodynamic properties like persistent currents. It was earlier verified for simple one dimensional systems. We study this formula very carefully and conclude that it requires reinterpretation in quasi one dimension. This is essentially because of the presence of evanescent modes in quasi one dimension.

PACS numbers:

I. INTRODUCTION

Due to the technological advances in nano-fabrication, it is possible to realize such small systems that the quantum mechanical coherence length of the electron extends through out the length of the sample. Quantum interference phenomena strongly determines the thermodynamic and transport properties of these so called mesoscopic systems¹.

Mesoscopic phenomenon can occur in canonical systems as well as in grand canonical systems. Mesoscopic systems are so small that even measuring probes (like voltage probe and current probe) can make the system a grand canonical system². A canonical mesoscopic system is well described by the Hamiltonian of the isolated system but that is not the case for grand canonical mesoscopic systems. A grand canonical mesoscopic system, by definition, is coupled to a reservoir with which it can exchange electrons. Mesoscopic systems are so small that the reservoir can drastically change the states of the system and this has to be explicitly accounted for³. One can take this into account by solving the Schrodinger equation of the leads and the system as a scattering problem. This approach is essentially known as Landauer-Butikker formalism. This formalism is thus different from the way we deal with large grand-canonical systems with the help of the grand partition function.

For example, if an Aharonov Bohm flux is applied through the center of a ring, the ring gets magnetized and a persistent current is generated in the ring⁴. This current arises because of the vector potential that changes the phase of the wave function in the ring and is another form of Aharonov Bohm effect which is an interference phenomenon, whereas the magnetization is a thermodynamic property is a consequence of that. One can cite many similar phenomenon¹. Persistent currents has been studied for more than a decade, theoretically as well as experimentally. If the ring is isolated then the persistent current is carried in some eigen-states. Whereas if it is open and connected to reservoirs, then persistent current is carried in resonant and non-resonant states that are typical of scattering states. Persistent current has been studied in several such grand canonical systems like a ring connected to a single reservoir or to many reservoirs⁵. If it is connected to many reservoirs at different chemical potentials, then non-equilibrium currents can co-exist with equilibrium persistent currents. In such open systems several interesting effects have been predicted, like current magnification in the presence

of transport^{6,7,8}, directional dependence of persistent currents⁹, current magnification effect in equilibrium systems in absence of transport current¹⁰, etc.

In order to realize a mesoscopic grand canonical system we connect the ring to reservoirs that are at fixed chemical potentials as is schematically shown in Fig. 1. The left reservoir has a chemical potential μ_1 and the right one has a chemical potential μ_2 . The reservoirs can also be at a finite temperature T . The ring is threaded by an Aharonov-Bohm flux. If $\mu_1 > \mu_2$, then there is a transport current (which is a non-equilibrium current) through the regions I and II. There is however no transport current in the ring. The ring will carry a persistent current which is an equilibrium current. Thus in the present geometry, the equilibrium persistent currents and non-equilibrium transport currents are spatially separated.

The Landauer-Buttiker approach proposes that an equilibrium phenomenon like persistent current in such an open system as that in Fig. 1, can be obtained from solving the scattering problem³. Akkermans et al¹¹ related the persistent current IS to the S matrix by the following formula.

$$IS = \frac{1}{2\pi i} \frac{\partial \log[\det(S)]}{\partial \phi} \quad (1)$$

Such a simple mathematical relation between the persistent current inside the ring and the S -matrix obtained from the wave-function far away from the ring is rather novel and resulted in a flurry of theoretical activities¹². While it is established (from theoretical and experimental point of view) that conductance (a non-equilibrium phenomenon) can be obtained from the S -matrix, Akkermans approach may prove to be the first step to obtain any equilibrium phenomenon from the S -matrix, that is a step towards a mesoscopic version of fluctuation dissipation theorem¹². The correctness of Eq. 1 has been explicitly verified in one dimension (1D) but not in quasi one dimension (Q1D). That complexities arise in Q1D due to the presence of evanescent modes has been observed recently^{13,14} although it is not very well known to the community¹⁵. For example Friedel sum rule¹³ and Buttiker-Thomas-Pertre formula¹⁴ breaks down in the presence of evanescent modes. So in this work we undertake the task of verifying if Akkermans formula is valid in Q1D. In case of Akkerman's formula, in this paper, we can show analytically how the evanescent modes complicates things. Earlier works^{13,14} on different formulas are essentially numerical verifications. Persistent current in the geometry of Fig. 1 has been studied earlier,

but always using the wave function. The S-matrix was never used and comparison was not made between Akkerman's approach and the usual wave function approach. This is done explicitly for the first time in Q1D in this work to show that the 1D result cannot be extended to Q1D due to the presence of evanescent modes.

II. MODEL AND METHOD

As shown in Fig. 1, we consider a ring coupled to a wire. The scattering solution for this geometry is discussed in detail in our earlier work⁵. Here we outline some points with respect to calculating the RHS of Eq. 1 which was not done earlier. There is a δ -potential impurity present in the ring at any arbitrary position X [Fig 1]. We apply Aharonov-Bohm flux ϕ through the ring, perpendicularly to the plane of the paper. We consider two modes of propagation because it will show the shortcomings of Akkerman's formula and that can be generalized analytically to any number of modes. The Schrödinger equation for a Q1D wire in presence of a δ -potential at $x = 0$, $y = y_i$ is (the third degree of freedom, i.e. z -direction, is usually frozen by creating a strong quantization¹)

$$\left[-\frac{\hbar^2}{2m^*}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + V_c(y)\right]\Psi(x, y) = E\Psi(x, y) \quad (2)$$

where the x -coordinate is along the wire and the y -coordinate is perpendicular to the wire. Here m^* is the electron mass and E is the electron energy. The wave-function in a ring can be obtained by solving the above equation with periodic boundary condition where we assume the ring to be so large that its curvature can be neglected. Here $V_c(y)$ is the confinement potential making up the quantum wires in Figure 1. The magnetic field just appears as a phase of $\Psi(x, y)$ that will be accounted for while applying boundary conditions. Eqn. 2 can be separated as

$$-\frac{\hbar^2}{2m^*}\frac{d^2\psi(x)}{dx^2} = \frac{\hbar^2 k^2}{2m^*}\psi(x) \quad (3)$$

and

$$\left[-\frac{\hbar^2}{2m^*}\frac{d^2}{dy^2} + V_c(y)\right]\chi_n(y) = E_n\chi_n(y) \quad (4)$$

Here we consider that electron is propagating along x direction. This means in regions *I* and *II* of Fig. 1, x direction is along the arrows. In region *III*, the x direction is along the line joining

P and Q. And in regions *IV* and *V*, the x direction is along the perimeter of the ring. One can choose different axes in the different regions as the matrix equations for mode matching is independent of this choice⁵. The confinement potential $V_c(y)$ in different regions is then in the y (transverse) direction. It can be seen from Eqs. 3 and 4 that

$$E = E_n + \frac{\hbar^2 k^2}{2m^*} \quad (5)$$

We take $V_c(y)$ to be a square well potential of width W , that gives $\chi_n(y) = \sin[\frac{n\pi}{W}(y + \frac{W}{2})]$. So $E_n = \frac{\hbar^2 n^2 \pi^2}{2m^* W^2}$. Hence, in the first mode,

$$k_1 = \sqrt{\frac{2m^* E}{\hbar^2} - \frac{\pi^2}{W^2}} \quad (6)$$

is the propagating wave-vector and in the second mode

$$k_2 = \sqrt{\frac{2m^* E}{\hbar^2} - \frac{4\pi^2}{W^2}} \quad (7)$$

is the propagating wave-vector. We have chosen $2m^* = 1$ and $\hbar = 1$.

When electrons are incident along region *I* (in Fig 1) in the first mode the scattering problem can be solved exactly. The solution to Eqn. 3 in region *I* becomes

$$\psi_I = \frac{1}{\sqrt{k_1}} e^{ik_1 x} + \frac{r'_{11}}{\sqrt{k_1}} e^{-ik_1 x} + \frac{r'_{12}}{\sqrt{k_2}} e^{-ik_2 x} \quad (8)$$

Similarly, in region *II*, *III*, *IV* and *V* we get

$$\psi_{II} = \frac{g'_{11}}{\sqrt{k_1}} e^{ik_1 x} + \frac{g'_{12}}{\sqrt{k_2}} e^{ik_2 x} \quad (9)$$

$$\psi_{III} = \frac{Ae^{ik_1 x}}{\sqrt{k_1}} + \frac{Be^{-ik_1 x}}{\sqrt{k_1}} + \frac{Ce^{ik_2 x}}{\sqrt{k_2}} + \frac{De^{-ik_2 x}}{\sqrt{k_2}} \quad (10)$$

$$\psi_{IV} = \frac{Ee^{ik_1 x}}{\sqrt{k_1}} + \frac{Fe^{-ik_1 x}}{\sqrt{k_1}} + \frac{Ge^{ik_2 x}}{\sqrt{k_2}} + \frac{He^{-ik_2 x}}{\sqrt{k_2}} \quad (11)$$

$$\psi_V = \frac{Je^{ik_1(x-l_2)}}{\sqrt{k_1}} + \frac{Ke^{-ik_1(x-l_2)}}{\sqrt{k_1}} + \frac{Le^{ik_2(x-l_2)}}{\sqrt{k_2}} + \frac{Me^{-ik_2(x-l_2)}}{\sqrt{k_2}} \quad (12)$$

where r'_{11} , r'_{12} , g'_{11} and g'_{12} , A , B , C , D , E , F , G , H , J , K , L and M are to be determined by mode matching.

Note that at P and Q we have a three legged junction that is schematically shown in Fig. 2. In a previous work⁵ we proposed a form of junction scattering matrix S_J for a two channel

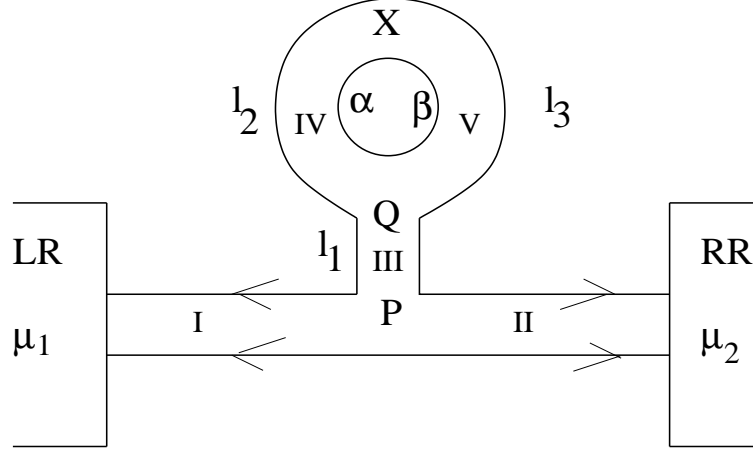


FIG. 1: A ring connected to an infinite wire. A δ function potential is present in the ring at position X . A chemical potential difference ($\mu_1 - \mu_2$) between the left reservoir (LR) and the right reservoir (RR) drives a transport current through the regions I and II. The ring is pierced by an Aharonov-Bohm flux that drives an equilibrium current called persistent current in the ring.

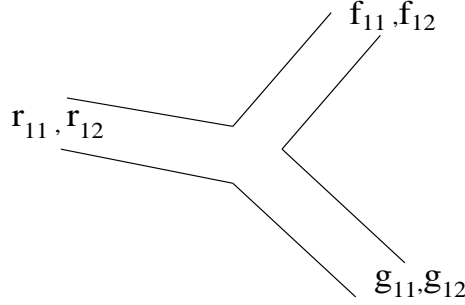


FIG. 2: A 3-leg junction.

quantum wire that can be easily generalized to any number of channels. For the δ potential impurity at X we use the scattering matrix S_b that was derived by Bagwell¹⁶. One can match the wavefunctions and conserve the currents by using these S -matrices that give us a set of linear equations. We calculate the coefficients $A, B, C, D, E, F, G, H, J, K, L$ and M numerically by matrix inversion.

Persistent current can be computed from the wave-function.

$$IW^{(k_1)} = \int_{-\frac{W}{2}}^{\frac{W}{2}} \frac{\hbar}{2im^*} (\psi^\dagger \vec{\nabla} \psi - \psi \vec{\nabla} \psi^\dagger) dy \quad (13)$$

Here the index k_1 implies that this is the current due to an incident electron in k_1 channel on the left. Similarly currents are generated due to incident electron in k_1 channel on the right, k_2 channel on the left and k_2 channel on the right. So the net observable persistent current is

$$IW = 2IW^{(k_1)} + 2IW^{(k_2)} \quad (14)$$

From Eq. 13 we get

$$IW^{(k_1)} = 2I_0(|E|^2 - |F|^2 + |G|^2 - |H|^2)^{(k_1)} \quad (15)$$

where $I_0 = \frac{\hbar e}{2m^*W^2}$.

We also calculate the scattering matrix elements $r'_{11}, r'_{12}, g'_{11}, g'_{12}, r'_{22}, r'_{21}, g'_{22}, g'_{21}$ by matrix inversion to form the scattering matrix of the system as

$$S = \begin{pmatrix} r'_{11} & r'_{12} & g'_{11} & g'_{12} \\ r'_{21} & r'_{22} & g'_{21} & g'_{22} \\ g'_{11} & g'_{12} & r'_{11} & r'_{12} \\ g'_{21} & g'_{22} & r'_{21} & r'_{22} \end{pmatrix} \quad (16)$$

Substituting 16 in 1, Eq. 1 too can be written as a sum of four terms¹⁷, where each term consist of scattering matrix elements due to incidence in a particular momentum channel. That is

$$IS = 2IS^{(k_1)} + 2IS^{(k_2)} \quad (17)$$

where

$$IS^{(k_1)} = \frac{1}{2\pi}(|r'_{11}|^2 \frac{\partial \arg(r'_{11})}{\partial \phi} + |r'_{12}|^2 \frac{\partial \arg(r'_{12})}{\partial \phi} + |g'_{11}|^2 \frac{\partial \arg(g'_{11})}{\partial \phi} + |g'_{12}|^2 \frac{\partial \arg(g'_{12})}{\partial \phi}) \quad (18)$$

and

$$IS^{(k_2)} = \frac{1}{2\pi}(|r'_{21}|^2 \frac{\partial \arg(r'_{21})}{\partial \phi} + |r'_{22}|^2 \frac{\partial \arg(r'_{22})}{\partial \phi} + |g'_{21}|^2 \frac{\partial \arg(g'_{21})}{\partial \phi} + |g'_{22}|^2 \frac{\partial \arg(g'_{22})}{\partial \phi}) \quad (19)$$

Although not implied by the notation, the currents defined above (Eqs. 14 and 17) are actually differential currents in an infinitesimal energy range dE . The integration of these will give the actual measurable currents. The integration limits depend on the chemical potential μ_1 and μ_2 . Temperature can be included through Fermi function. All expressions so far is derived for both modes being propagating. Earlier it was shown that $IW = IS$ for one dimensional ring coupled to a reservoir¹¹. We shall show below that when all modes are propagating then one gets $IS = IW$, but not when we include evanescent modes. This is because when evanescent modes are present then some expressions can be analytically continued to include evanescent modes but not all of them.

III. INCLUSION OF EVANESCENT MODES

E is the energy of incidence that can be varied as an external parameter by tuning the chemical potentials of the reservoirs. When $\pi^2 \leq 2m^*EW^2/\hbar^2 < 4\pi^2$, then it can be seen from Eq. 7 and Eqs. 8-12 that k_2 mode becomes evanescent. Eqs. 8-12 are still solutions to Schrodinger Eq. 3 implying electrons in the ring can be coupled to an evanescent channel due to scattering¹⁶. A single impurity can couple an electron to the evanescent second channel. Scattering at the junctions can also couple an electron to the evanescent second channel. No electron can be incident from (or emitted to) ∞ along the evanescent 2nd channel. So the scattering problem has to be solved with an incident electron in k_1 mode in the left lead and an outgoing electron in k_1 mode in the right lead. Hence the S matrix becomes 2×2 and is given by

$$S = \begin{pmatrix} r'_{11} & g'_{11} \\ g'_{11} & r'_{11} \end{pmatrix} \quad (20)$$

Although the S matrix is 2×2 , its calculation has to be done by using the 6×6 junction S-matrix S_J that is defined in Ref. [5] and the 4×4 S-matrix S_b for the δ function potential that is defined in Ref. [16]. This is essential because evanescent modes can be coupled inside the ring without violating any physical principle like conservation of energy. g'_{12} , r'_{12} etc are also non-zero, but they do not carry any current. They are not S-matrix elements any more. Rather they define the coupling to evanescent modes. Unitarity should imply $|r'_{11}|^2 + |g'_{11}|^2 = 1$ and indeed

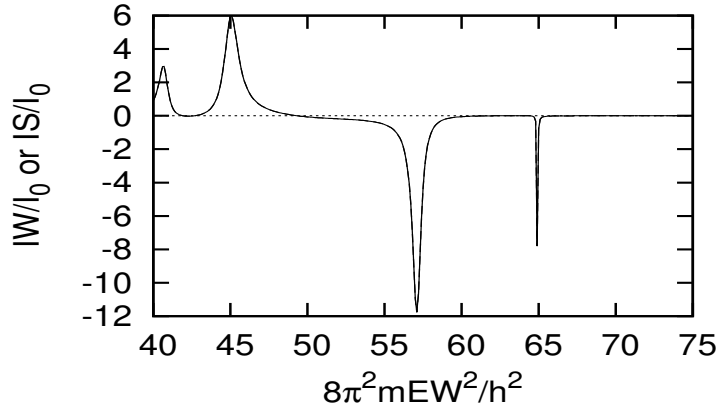


FIG. 3: IW/I_0 and IS/I_0 vs $8\pi^2 m^* E W^2 / \hbar^2$. The system parameters are $l_1 = l_2 = l_3 = 1$, $y_i = 0.1$, $\alpha = \beta = 0.3$ and $\gamma = 4$.

we get this from the junction matrix defined by S_J . This implies that S_J is appropriate for accounting for realistic multichannel situations.

IV. RESULTS AND DISCUSSIONS

In a real system, there are always propagating modes as well as evanescent modes. As is evident from Eqs. 6 and 7, evanescent modes have higher transverse energy than the propagating modes. Also higher the n value of the evanescent mode, higher is its transverse energy. This energy is at the cost of the propagation energy which becomes more and more negative for higher n evanescent modes. There will be a natural cut off as very high energies cannot be realized in a quantum wire. In our simplistic approach we will first consider a case when there are two modes in the wire, both being propagating. We will then consider a case when one mode is propagating and the other is evanescent. In the first case we will verify that Akkerman's approach gives exactly the same result as the current calculated from the wave-function (i.e., $IS = IW$). In the second case there are complexities. It will be argued that such complexities will persist in a real system where there will be many evanescent modes.

When both modes are propagating: First we consider the energy range $4\pi^2 \leq 2m^* E W^2 / \hbar^2 \leq 9\pi^2$ (i.e. $39.478 \leq 2m^* E W^2 / \hbar^2 \leq 88.826$). Substituting this E in Eqs. 6 and 7 we can see that

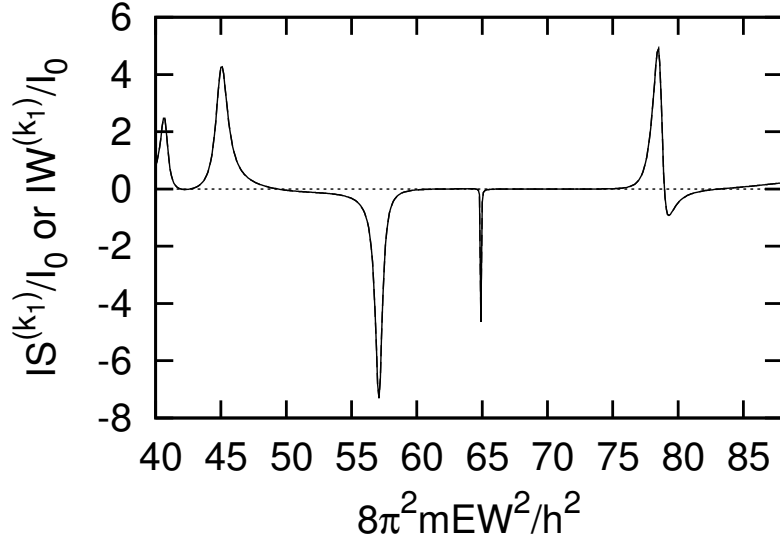


FIG. 4: $IS^{(k_1)}/I_0$ and $IW^{(k_1)}/I_0$ versus $8\pi^2 m^* E W^2 / \hbar^2$. The system parameters are $l_1 = l_2 = l_3 = 1$, $y_i = 0.1$, $\alpha = \beta = 0.3$, $\gamma = 4$.

both the modes are propagating. The nature of current is plotted in Fig 3. The figure shows that the current IS obtained from Akkerman's formula (that is from S -matrix) is identical with IW (that is from wave function). In fact, $IS^{(k_1)}$ and $IS^{(k_2)}$ are individually identical with $IW^{(k_1)}$ and $IW^{(k_2)}$, respectively. These are shown in Fig. 4 and Fig. 5. This implies that $IW^{(k_1)}$ is the same algebraic expression as $IS^{(k_1)}$. Similarly for $IW^{(k_2)}$ and $IS^{(k_2)}$. And also IS and IW are the same algebraic expressions.

When one mode is evanescent: Now consider the energy range $\pi^2 \leq 2m^* E W^2 / \hbar^2 < 4\pi^2$ (i.e. $9.87 \leq 2m^* E W^2 / \hbar^2 \leq 39.477$) so that $k_2 = \sqrt{\frac{2m^* E}{\hbar^2} - \frac{4\pi^2}{W^2}}$ becomes imaginary ($k_2 \rightarrow i\kappa_2$) while $k_1 = \sqrt{\frac{2m^* E}{\hbar^2} - \frac{\pi^2}{W^2}}$ remains real. In this regime the ring contains one propagating and one evanescent mode. Evanescent mode current can be calculated by directly applying Eqn. 13 to evanescent mode wave functions or it can be calculated by analytically continuing propagating mode current 14 to below the barrier (that is $k_2 \rightarrow i\kappa_2$). We have already argued that IS and IW are the same algebraic expression. Under the transformation $k_2 \rightarrow i\kappa_2$, applied to both IS and IW , they definitely remain the same algebraic expression. However, Akkerman's formula takes a different meaning in this regime where there are evanescent modes. This is essential

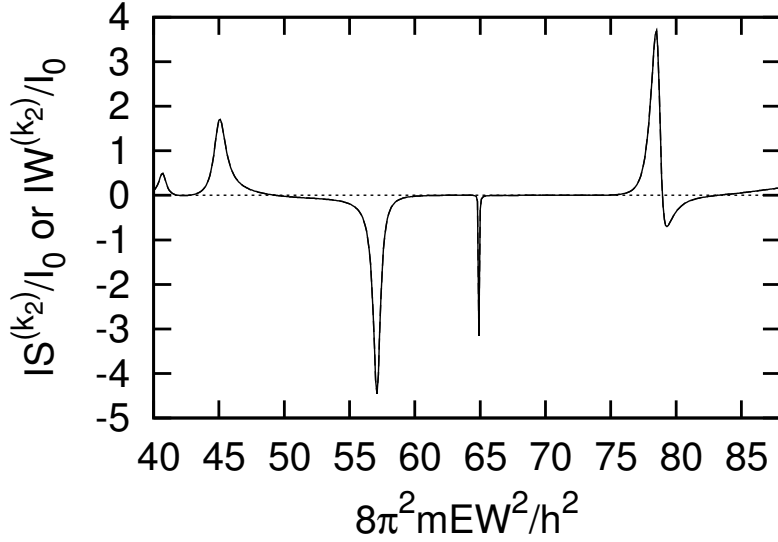


FIG. 5: $IS^{(k_2)}/I_0$ and $IW^{(k_2)}/I_0$ versus $8\pi^2 m^* E W^2 / h^2$. The system parameters are $l_1 = l_2 = l_3 = 1$, $y_i = 0.1$, $\alpha = \beta = 0.3$, $\gamma = 4$.

because the Akkerman's formula in Eq. 1 is related to the S-matrix and the transformed expression ($IS_{k_2 \rightarrow i\kappa_2}$) cannot be obtained from the S-matrix. We know that no electron can be incident along the evanescent channel. So $IW^{(k_2)}$ and $IS^{(k_2)}$ are zero. So IW is now just equal to $IW^{(k_1)}$, where k_2 has been analytically continued. If one assumes that in this regime IS is equal to $IS^{(k_1)}$ where k_2 is analytically continued, then obviously once again $IS = IW$, as they are the same algebraic expression. However, one can see that $IS^{(k_1)}$ (see Eq. 18) cannot be obtained by substituting the S-matrix (Eq. 20), into Akkerman's formula (Eq. 1). If we do this substitution, then we will get the first term and the third term in Eq. 18 (where of course $k_2 \rightarrow i\kappa_2$ transformation is taken care of). We will not get the 2nd and 4th terms as r'_{12} and g'_{12} are not S-matrix elements any more. In fact, we see from Fig 6 that the difference between IS and IW is quite large.

Since IS is now reduced to just two terms, one may ask the question that will it give the partial current in the propagating channel only. Because this partial current also consists of two terms only. Note from Eqn. 15 that the total current $IW^{(k_1)} = IW_1^{(k_1)} + IW_2^{(k_1)}$, where $IW_1^{(k_1)} = 2I_0(|E|^2 - |F|^2)^{(k_1)}$, E and F being the wave function amplitudes in the

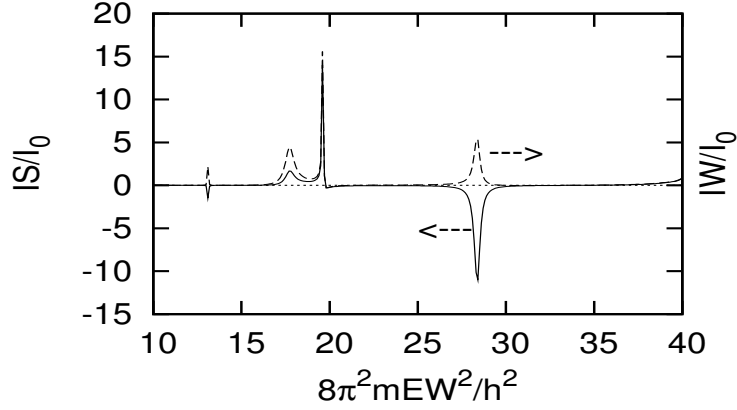


FIG. 6: IS/I_0 (solid line) and IW/I_0 (dashed line) versus $8\pi^2 m^* EW^2/h^2$ of the system when second channel is evanescent. The system parameters are $l_1 = l_2 = l_3 = 1$, $y_i = 0.1$, $\alpha = \beta = 0.3$, $\gamma = 4$. Here IS/I_0 is obtained by substituting S given in Eq. 20 into Eq. 1.

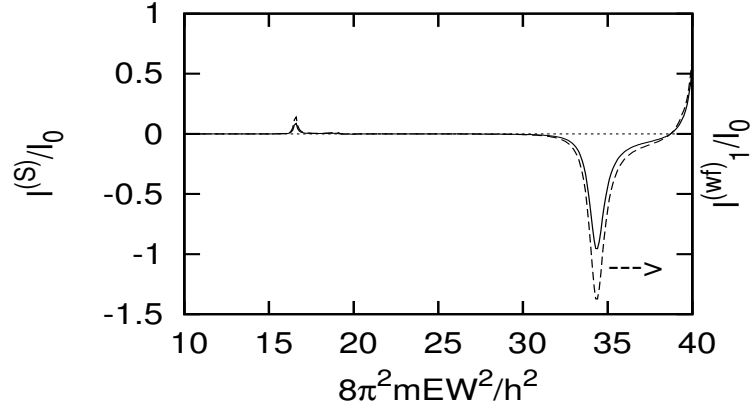


FIG. 7: IS/I_0 (solid line) and $IW_1^{(k_1)}/I_0$ (dashed line) versus $8\pi^2 m^* EW^2/h^2$. The system parameters are $l_1 = l_2 = l_3 = 1$, $y_i = 0.1$, $\alpha = \beta = 0.3$, $\gamma = -3.7$. Here IS/I_0 is obtained by substituting S given in Eq. 20 into Eq. 1.

propagating channel and $IW_2^{(k_1)} = 2I_0(|G|^2 - |H|^2)^{(k_1)}$, G and H being the wave function amplitudes in the evanescent channel. We have plotted $IW_1^{(k_1)}$ in Fig. 7. The figure shows that IS differs from $IW_1^{(k_1)}$. So it is confirmed that IS obtained from Eq. 1 neither give the total measurable current of the system nor the partial current through the propagating channel.

V. CONCLUSIONS

For realistic mesoscopic rings connected to leads, there are always evanescent modes. The S-matrix is always defined by the propagating modes only. For such systems one cannot directly apply Akkerman's formula. Instead one should start with a model where all the modes are made propagating. One should apply Akkerman's formula to the S-matrix of this system and then analytically continue this expression for the current to the situation where an appropriate number of modes are evanescent. While the Landauer-Buttiker approach is still inevitable as the evanescent modes are obtained due to an incident electron that is scattered to evanescent modes, the formula given in Eq. 1 is no longer strictly valid in presence of evanescent modes.

VI. ACKNOWLEDGMENT

One of us (PSD) would like to thank ICTP for local hospitality and other facilities where a part of this work was done.

-
- ¹ S. Datta, Electronic transport in mesoscopic systems, Cambridge university press, 1995.
- ² M. Buttiker, Phys. Rev. Lett. **57**, 1761 (1986)
- ³ M. Buttiker, Phys. Rev. B **32**, 1846-1849 (1985).
- ⁴ M. Buttiker, Y. Imry and R. Landauer, Phys. Lett. A **96**, 365 (1983).
- ⁵ S. Sengupta Chowdhury, P. Singha Deo, A.K. Roy and M. Manninen, New Journal of Phys. **10**, 1 (2008).
- ⁶ A. M. Jayannavar and P. Singha Deo Phys. Rev. B **51**, 10175(1995).
- ⁷ T. P. Pareek, P. Singha Deo and A. M. Jayannavar Phys. Rev. B. **52**, 14657 (1995).
- ⁸ S. Bandopadhyay, P. Singha Deo and A.M. Jayannavar Phys. Rev. B **70**, 075315 (2004).
- ⁹ A. M. Jayannavar and P. Singha Deo Phys. Rev. B **49**, 13685 (1994).
- ¹⁰ Colin Benjamin, A. M. Jayannavar, Phys. Rev. B **64**, 233406 (2001).
- ¹¹ E. Akkermans, A. Auerbach, J. E. Avron, and B. Shapiro Phys. Rev. Lett. **66**, 76 (1991)
- ¹² E. Akkermans and G. Montambaux, Phys. Rev. Lett. **68**, 642 (1992).

- ¹³ Swarnali Bandopadhyay and P. Singha Deo, Phys. Rev. B **68** 113301 (2003)
- ¹⁴ P. Singha Deo, International Journal of Modern Physics B, **19**, 899 (2005)
- ¹⁵ D.A. Abanin and L.S. Levitov, Phys. Rev. Lett. **93**, 126802 (04).
- ¹⁶ P.F.Bagwell, Phys. Rev.B **41**, 10354(1990).
- ¹⁷ Dashen, Ma and Bernstein, Phys. Rev. **187**, 345 (1969); M. Buttiker, Pramana Journal of Physics,**58**, 241 (2002).